Asymptotic entanglement of two atoms in squeezed light field

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The dynamics of entanglement between two - level atoms interacting with a common squeezed reservoir is investigated. It is shown that for spatially separated atoms there is a unique asymptotic state depending on the distance between the atoms and the atom - photons detuning. In the regime of strong correlations there is a one - parameter family of asymptotic steady - states depending on initial conditions. In contrast to the thermal reservoir both types of asymptotic states can be entangled. We calculate the amount of entanglement in the system in terms of concurrence.

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I. INTRODUCTION

Dynamical creation of entanglement by the indirect interaction between otherwise decoupled systems has been recently studied by many researchers mainly in the case of two - level atoms interacting with the common vacuum. The idea that dissipation can create rather then destroy entanglement was put forward in several publications [1–4]. In particular, the effect of spontaneous emission on the destruction and production of entanglement was discussed [5–8]. When the two atoms are separated by a small distance compared to the radiation wavelength, there is a substantial probability that a photon emitted by one atom will be absorbed by the other, and the resulting process of photon exchange produces correlations between the atoms. Such correlations may cause that initially separable states become entangled.

The case of two atoms immersed in a common thermal reservoir was also investigated [9–12]. As was shown in [12], similarly to the vacuum case, the collective properties of the atomic system can alter the decay process compared to the single atom. There are states with enhanced emission rates and such that the emission rate is reduced. The important example of the latter is the antisymmetric superposition $|a\rangle$ constructed from energy levels of considered atoms. When the atoms are close to each other, this state is decoupled from the environment and therefore is stable. In that case the asymptotic states of the system are parametrized by the fidelity F of the initial state with respect to the state $|a\rangle$ and the temperature T of the photon reservoir. Moreover, the asymptotic states can be identified with thermal generalization of Werner states i.e. mixtures of the state $|a\rangle$ and Gibbs equilibrium state at the temperature T.

In the present paper, we consider the atoms interacting with photon reservoir in a squeezed state [13]. In practice, squeezed light sources produce photon fields in multimode squeezed states, but here we assume broadband approximation in which the parameters characterizing the photon field are constant over a sufficiently broad frequency range. The dynamics of atoms interacting with squeezed light was studied by many authors (see e.g. review paper [14] and references therein). In the context of our studies, we mention the result of Palma and Knight [15] showing the existence of highly correlated asymptotic state and the analysis of cooperative behavior of atoms in broadband squeezed light in Ref. [16].

In this paper, we study the asymptotic entanglement of the system of atoms evolving according the master equation considered by Tanaś and Ficek [18] but we allow non-zero detuning between the atomic transition frequency and the carrier frequency of the photon field. In the case of spatially separated atoms studied in details in Ref. [18], there exists a unique asymptotic state, but in contrast to the vacuum or thermal reservoirs, this state can be entangled. However, the produced entanglement is maximal only when the atoms are in resonance with the squeezed photon field. Non - zero detuning significantly diminishes this production. When the atoms are close to each other, the dynamics of the system radically changes. As in the vacuum or thermal case, the antisymmetric state again decouples from the reservoir, therefore is stable. The asymptotic states ρ_{as} depend on the initial fidelity F and parameters describing the reservoir, but non - zero detuning also modifies the matrix elements of ρ_{as}. We show that the asymptotic states can be expressed as a mixture of a separable Gibbs state and two pure entangled states: an antisymmetric state $|a\rangle$ and some symmetric superposition of ground and excited levels of the atoms. This realization of the asymptotic state simplifies for zero detuning and minimum - uncertainty squeezed reservoir to the mixture of $|a\rangle$ and two - atom squeezed state [15]. Thus in that case, there are two linearly

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independent stable pure states so that decoherence - free subspace is two - dimensional [17].

Depending on the initial fidelity, some of the asymptotic states are entangled. We calculate the amount of the asymptotic entanglement using the concurrence as its measure. We also show that for initial fidelity greater than some threshold value (depending on the properties of the reservoir and detuning), the asymptotic concurrence is non - zero. This property is analogous to the thermal reservoir case. But when the reservoir is in a squeezed state, somehow unexpected result occurs: initial states with small or even zero fidelity become asymptotically entangled. The possibility of production of entanglement starting from separable states with zero fidelity is very interesting. In this case the correlations present in a squeezed reservoir are transferred to the atomic system, entangling for example two atoms both in the ground state. But as before, large detuning between atoms and photon field, destroys this possibility.

II. MODEL DYNAMICS

Consider two-level atoms A and B with ground states $|0\rangle_j$ and excited states $|1\rangle_j$ ($j=A,B$), interacting with the radiation field in a broadband squeezed vacuum state with the carrier frequency ω_s. The parameters N and M characterizing the squeezing satisfy

$$M=|M|\,e^{i\vartheta}\quad\text{and}\quad |M|\leq\sqrt{N(N+1)},$$

where the equality holds for a minimum-uncertainty squeezed state. In the Markov approximation the influence of the reservoir on the system of atoms can be described by the dynamical semi-group with the Lindblad generator [18]

$$L=-i[H,\cdot]+L_{\mathrm{D}},$$

where

$$H=\frac{\omega_0}{2}\sum_{j=A,B}\sigma_3^j+\sum_{\substack{j,k=A,B\\ j\neq k}}\Omega_{jk}\sigma_+^j\sigma_-^k, \tag{II.1}$$

and

$$\begin{aligned}L_{\mathrm{D}}\rho=&\frac{1}{2}\sum_{j,k=A,B}\gamma_{jk}(1+N)\left(2\sigma_-^j\rho\sigma_+^k-\sigma_+^k\sigma_-^j\rho-\rho\sigma_+^k\sigma_-^j\right)\\ &+\frac{1}{2}\sum_{j,k=A,B}\gamma_{jk}N\left(2\sigma_+^j\rho\sigma_-^k-\sigma_-^k\sigma_+^j\rho-\rho\sigma_-^k\sigma_+^j\right)\\ &+\frac{1}{2}\sum_{j,k=A,B}\gamma_{jk}M\left(2\sigma_+^j\rho\sigma_+^k-\sigma_+^k\sigma_+^j\rho-\rho\sigma_+^k\sigma_+^j\right)e^{-2i\omega_s t}\\ &+\frac{1}{2}\sum_{j,k=A,B}\gamma_{jk}\overline{M}\left(2\sigma_-^j\rho\sigma_-^k-\sigma_-^k\sigma_-^j\rho-\rho\sigma_-^k\sigma_-^j\right)e^{2i\omega_s t}.\end{aligned} \tag{II.2}$$

Here

$$\sigma_\pm^A=\sigma_\pm\otimes\mathbb{1},\quad \sigma_\pm^B=\mathbb{1}\otimes\sigma_\pm,\quad \sigma_3^A=\sigma_3\otimes\mathbb{1},\quad \sigma_3^B=\mathbb{1}\otimes\sigma_3.$$

In the Hamiltonian (II.1), ω_0 is the frequency of the transition $|0\rangle_j\to|1\rangle_j$ ($j=A,B$) and $\Omega_{AB}=\Omega_{BA}=\Omega$ describes interatomic coupling by the dipole-dipole interaction. On the other hand, dissipative dynamics is given by the generator (II.2) with parameters γ_{AB} satisfying

$$\gamma_{AA}=\gamma_{BB}=\gamma_0,\quad \gamma_{AB}=\gamma_{BA}=\gamma. \tag{II.3}$$

In the above equalities, γ_0 is the single atom spontaneous emission rate, and $\gamma=G(\vec{r}_{AB})\,\gamma_0$ is the collective damping constant. In the model considered, $G(\vec{r}_{AB})$ is the function of the interatomic distance $\vec{r}_{AB}$, and $G(\vec{r}_{AB})$ is small for large separation of atoms. On the other hand, $G(\vec{r}_{AB})\to 1$ when $\vec{r}_{AB}$ is small (for more details see e.g. [19]).

The time evolution of the system of atoms is given by the master equation

$$\frac{d\rho}{dt}=L\,\rho, \tag{II.4}$$

In the frame rotating with frequency ω_s, the master equation (II.4) becomes an equation with time indendent coefficients, and it may be written as

$$\frac{d\rho_I}{dt}=\widetilde{L}\,\rho_I, \tag{II.5}$$

where

$$\widetilde{L}=-i[\widetilde{H},\cdot]+\widetilde{L}_{\mathrm{D}},$$

with

$$\widetilde{H}=\frac{\delta_0}{2}\sum_{j=A,B}\sigma_3^j+\sum_{\substack{j,k=A,B\\ j\neq k}}\Omega_{jk}\sigma_+^j\sigma_-^k,\quad \delta_0=\omega_0-\omega_s, \tag{II.6}$$

and

$$\begin{aligned}\widetilde{L}_{\mathrm{D}}\rho_I=&\frac{1}{2}\sum_{j,k=A,B}\gamma_{jk}(1+N)\left(2\sigma_-^j\rho_I\sigma_+^k-\sigma_+^k\sigma_-^j\rho_I-\rho_I\sigma_+^k\sigma_-^j\right)\\ &+\frac{1}{2}\sum_{j,k=A,B}\gamma_{jk}N\left(2\sigma_+^j\rho_I\sigma_-^k-\sigma_-^k\sigma_+^j\rho_I-\rho_I\sigma_-^k\sigma_+^j\right)\\ &+\frac{1}{2}\sum_{j,k=A,B}\gamma_{jk}M\left(2\sigma_+^j\rho_I\sigma_+^k-\sigma_+^k\sigma_+^j\rho_I-\rho_I\sigma_+^k\sigma_+^j\right)\\ &+\frac{1}{2}\sum_{j,k=A,B}\gamma_{jk}\overline{M}\left(2\sigma_-^j\rho_I\sigma_-^k-\sigma_-^k\sigma_-^j\rho_I-\rho_I\sigma_-^k\sigma_-^j\right).\end{aligned} \tag{II.7}$$

Notice that in the Hamiltonian (II.6), detuning δ_0 can be arbitrary. Only when the atoms are in resonance with the carrier frequency of the squeezed vacuum, $\delta_0=0$.

From now on we omit the subscript I. The master equation (II.5) can be used to obtain the equations for matrix elements of a state ρ of the system of two-level atoms with respect to some basis. To simplify the calculations one can work in the basis of collective states in the Hilbert space $\mathbb{C}^2\otimes\mathbb{C}^2$ [19], given by product vectors

$$|e\rangle=|1\rangle_A\otimes|1\rangle_B,\quad |g\rangle=|0\rangle_A\otimes|0\rangle_B, \tag{II.8}$$

symmetric superposition

$$|s\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B\right), \tag{II.9}$$

and antisymmetric superposition

$$|a\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle_A \otimes |0\rangle_B - |0\rangle_A \otimes |1\rangle_B\right). \tag{II.10}$$

In the basis of collective states, two-atom system can be treated as a single four-level system with ground state $|g\rangle$, excited state $|e\rangle$ and two intermediate states $|s\rangle$ and $|a\rangle$. From (II.5) it follows that the matrix elements of the state ρ with respect to the basis $|e\rangle$, $|s\rangle$, $|a\rangle$, $|g\rangle$ satisfy the equations which can be grouped into decoupled systems of differential equations. So the diagonal matrix elements and ρ_{eg}, satisfy

$$\begin{aligned}
\frac{d\rho_{ee}}{dt} &= (\gamma_0-\gamma)N\rho_{aa} + (\gamma_0+\gamma)N\rho_{ss} - 2\gamma_0 N\rho_{ee} \\
&\quad - \gamma(M\rho_{ge} + \overline{M}\rho_{eg}), \\
\frac{d\rho_{ss}}{dt} &= -(\gamma_0+\gamma)[(1+2N)\rho_{ss} - (1+n)\rho_{ee} - N\rho_{gg} \\
&\quad - M\rho_{ge} - \overline{M}\rho_{eg}], \\
\frac{d\rho_{aa}}{dt} &= -(\gamma_0-\gamma)[(1+2N)\rho_{aa} - (1+N)\rho_{ee} - N\rho_{gg} \\
&\quad + M\rho_{ge} + \overline{M}\rho_{eg}], \\
\frac{d\rho_{gg}}{dt} &= (\gamma_0-\gamma)(1+N)\rho_{aa} + (\gamma_0+\gamma)(1+N)\rho_{ss} - 2\gamma_0 N\rho_{gg} \\
&\quad - \gamma(M\rho_{ge} + \overline{M}\rho_{eg}), \\
\frac{d\rho_{eg}}{dt} &= -(\gamma_0-\gamma)\rho_{aa} + (\gamma_0+\gamma)M\rho_{ss} - \gamma M\rho_{gg} \\
&\quad - (\gamma_0(1+2N) + 2i\delta_0)\rho_{eg}.
\end{aligned} \tag{II.11}$$

On the other hand, the elements ρ_{ae}, ρ_{ag}, ρ_{se} and ρ_{sg} are connected by the following equations

$$\begin{aligned}
\frac{d\rho_{ae}}{dt} &= \left[\gamma\left(N+\frac{1}{2}\right) - \gamma_0\left(2N+\frac{1}{2}\right) + i(\delta_0+\Omega)\right]\rho_{ae} \\
&\quad - (\gamma_0-\gamma)\rho_{ga} + (\gamma_0-\gamma)\overline{M}\rho_{ea} - \gamma\overline{M}\rho_{ag}, \\
\frac{d\rho_{ag}}{dt} &= \left[\gamma\left(N+\frac{1}{2}\right) - \gamma_0\left(2N+\frac{1}{2}\right) - i(\delta_0-\Omega)\right]\rho_{ag} \\
&\quad + (\gamma_0-\gamma)M\rho_{ga} - (\gamma_0-\gamma)(1+N)\rho_{ea} - \gamma M\rho_{ae}, \\
\frac{d\rho_{se}}{dt} &= -\left[\gamma\left(N+\frac{1}{2}\right) + \gamma_0\left(2N+\frac{1}{2}\right) - i(\delta_0-\Omega)\right]\rho_{se} \\
&\quad + (\gamma_0+\gamma)\overline{M}\rho_{es} + (\gamma_0+\gamma)N\rho_{gs} - \gamma\overline{M}\rho_{sg}, \\
\frac{d\rho_{sg}}{dt} &= -\left[\gamma\left(N+\frac{1}{2}\right) + \gamma_0\left(2N+\frac{1}{2}\right) + i(\delta_0+\Omega)\right]\rho_{se} \\
&\quad + (\gamma_0+\gamma)(1+N)\rho_{es} + (\gamma_0+\gamma)M\rho_{gs} - \gamma M\rho_{se},
\end{aligned} \tag{II.12}$$

and finally

$$\frac{d\rho_{as}}{dt} = -[\gamma_0(1+2N) - 2i\Omega]\rho_{as}. \tag{II.13}$$

The equations for the remaining matrix elements can be obtained by using hermiticity of ρ.

From the equations (II.11) it follows that similarly as in the case of reservoir in the vacuum state (see e.g. [19]) and thermal state [12], the system of atoms in the symmetric state $|s\rangle$ decays with the enhanced rate $\gamma_0+\gamma$, whereas antisymmetric initial state $|a\rangle$ leads to the reduced rate $\gamma_0-\gamma$. When the atoms are so close to each other that we can ignore the effects of their different spatial positions, we can put $\gamma=\gamma_0$. In this limiting case of strongly correlated atoms (Dicke model) the state $|a\rangle$ is completely decoupled from the reservoir. It can be also checked that the master equation (II.5) describes two types of time evolution of the system of atoms, depending on the relation between γ and γ_0. When $\gamma<\gamma_0$, there is a unique asymptotic state. This state was found in Ref. [18] for the special case of zero detuning. In general case we compute it in the next section. On the other hand, in the Dicke model case when $\gamma=\gamma_0$, we show that there is a one - parameter family of nontrivial asymptotic states depending on the initial states.

III. ASYMPTOTIC STATES

A. Spatially separated atoms

We start with the case of spatially separated atoms, when $\gamma<\gamma_0$. Direct calculations show that in that case, there exists a unique stationary asymptotic state ρ_{u}, which in the canonical basis

$$|1\rangle_A\otimes|1\rangle_B,\ |1\rangle_A\otimes|0\rangle_B,\ |0\rangle_A\otimes|1\rangle_B,\ |0\rangle_A\otimes|0\rangle_B$$

has non-vanishing matrix elements

$$\begin{aligned}
&\rho_{11} = \frac{a_0}{u_0}, \quad \rho_{22} = \rho_{33} = \frac{c_0}{u_0}, \\
&\rho_{23} = \frac{b_0}{u_0}, \quad \rho_{14} = \frac{z_0}{u_0}, \quad \rho_{44} = \frac{d_0}{u_0},
\end{aligned} \tag{III.1}$$

where for

$$\delta = \frac{\delta_0}{\gamma_0}, \quad \widehat{\gamma} = \frac{\gamma}{\gamma_0},$$

we have

$$\begin{aligned}
u_0 =& (1+2N)^2\left[(1+2N)^2 + 4\delta^2\right] \\
&+ 4|M|^2(\widehat{\gamma}^2 - (1+2N)^2),
\end{aligned} \tag{III.2}$$

and

$$\begin{aligned}
&a_0 = N^2\left[(1+2N)^2 - 4|M|^2 + 4\delta^2\right] + |M|^2\widehat{\gamma}^2, \\
&c_0 = N(N+1)\left[(1+2N)^2 - 4|M|^2 + 4\delta^2\right] + |M|^2\widehat{\gamma}^2, \\
&d_0 = (1+N)^2\left[(1+2N)^2 - 4|M|^2 + 4\delta^2\right] + |M|^2\widehat{\gamma}^2.
\end{aligned} \tag{III.3}$$

Moreover,

$$b_0 = -2\widehat{\gamma}|M|^2,\qquad z_0 = -(1+2N-2i\delta)\widehat{\gamma}M. \tag{III.4}$$

The state (III.1) in contrast to the analogous asymptotic state in the thermal reservoir, can be entangled and, as we show later, its entanglement crucially depends on the value of the normalized detuning δ and the normalized damping constant $\widehat{\gamma}$.

B. Strongly correlated atoms

When $\gamma = \gamma_0$, equations (II.11) - (II.13) simplify and one can check that the solutions of (II.12) and (II.13) asymptotically vanish and the only contribution to the asymptotic states $\rho_{\rm as}$ comes from $\rho_{ee}, \rho_{aa}, \rho_{ss}, \rho_{gg}$ and ρ_{eg}. Note that in this case

$$\frac{d\rho_{aa}}{dt} = 0, \quad \text{so} \quad \rho_{aa}(t) = \rho_{aa}(0) = F,$$

where

$$F = \langle a|\rho|a\rangle$$

is the *fidelity* of the initial state ρ with respect to the antisymmetric state $|a\rangle$. Hence the fidelity the asymptotic state $\rho_{\rm as}$ also equals to F and one finds that in the canonical basis the matrix of $\rho_{\rm as}$ has the same "X" form as in the case of the state (III.1), but with non - vanishing matrix elements given by

$$\begin{aligned}
\rho_{11} &= (1-F)\,\frac{a}{u},\\
\rho_{22} &= (1-F)\,\frac{c}{2u} + \frac{F}{2},\\
\rho_{23} &= (1-F)\,\frac{c}{2u} - \frac{F}{2},\\
\rho_{14} &= (1-F)\,\frac{z}{u},\\
\rho_{44} &= (1-F)\,\frac{d}{u},
\end{aligned} \tag{III.5}$$

and $\rho_{33} = \rho_{22}$. In the equations (III.5) we have

$$\begin{aligned}
u =&(1+2N)^2\,(1+3N+3N^2-3|M|^2)\\
&+4\,(1+3N+3N^2)\,\delta^2,
\end{aligned} \tag{III.6}$$

and

$$\begin{aligned}
a &= 4N^2\left[N(N+1)-|M|^2\right] + |M|^2\\
&\quad + N^2\,(1+4\delta^2),\\
c &= (1+2N)^2\left[N(N+1)-|M|^2\right]\\
&\quad + 2N(N+1)\delta^2,\\
d &= (1+2N)\left[1+N+3(N(N+1)-|M|^2)\right]\\
&\quad + 2N\left[N(N+1)-|M|^2\right] + 4(1+N)^2\delta^2\\
z &= -(1+2N-2i\delta)\,M.
\end{aligned} \tag{III.7}$$

The asymptotic states $\rho_{\rm as}$ defined by (III.5) exists for any initial state, and for fixed parameters characterizing the squeezing depend on the initial fidelity and the normalized detuning $\delta = \delta_0/\gamma_0$ of the electromagnetic field. When $M = 0$, we recover the case of standard thermal bath with N playing the role of the mean photon number [12].

To study the structure of the asymptotic states, we consider first the special case of a minimum-uncertainty squeezing and zero detuning of the radiation field. One can check that in that case, the matrix elements of $\rho_{\rm as}$ are given by

$$\begin{aligned}
\rho_{11} &= (1-F)\,\frac{N}{1+2N},\\
\rho_{22} &= (\rho_{\rm as})_{33} = \frac{F}{2},\\
\rho_{23} &= -\frac{F}{2},\\
\rho_{44} &= (1-F)\,\frac{1+N}{1+2N},\\
\rho_{14} &= (1-F)\,\frac{\sqrt{N(N+1)}}{1+2N}\,e^{i\theta},
\end{aligned} \tag{III.8}$$

where $\theta = \vartheta + \pi$. The asymptotic state given by (III.8) has a remarkable structure: it is a mixture

$$\rho_{\rm as} = (1-F)\,|N,\theta\rangle\langle N,\theta| + F\,|a\rangle\langle a| \tag{III.9}$$

of the pure state

$$|N,\theta\rangle = \sqrt{\frac{N}{1+2N}}\,|0\rangle_A\otimes|0\rangle_B + e^{i\theta}\sqrt{\frac{1+N}{1+2N}}\,|1\rangle_A\otimes|1\rangle_B \tag{III.10}$$

and the antisymmetric state $|a\rangle$. The state $|N,\theta\rangle$ is known as two - atom squeezed state, and can be obtained from the ground state $|g\rangle = |0\rangle_A\otimes|0\rangle_B$ by applying the atomic squeezing transformation $S(\xi)$, given by

$$S(\xi) = \exp\left(\overline{\xi}\,\sigma_-^A\sigma_-^B - \xi\,\sigma_+^A\sigma_+^B\right), \tag{III.11}$$

for the appropriate choice of the complex parameter ξ [15]. This state is entangled and in the limit of maximal squeezing ($N\to\infty$), it becomes a maximally entangled generalized Bell state. Notice also that $|a\rangle$ and $|N,\theta\rangle$ span decoherence - free subspace for this specific system, as it was recently established in Ref. [17].

In a general case the structure of $\rho_{\rm as}$ is much more involved. Define

$$F_{\rm cr} = \frac{c}{c+u}. \tag{III.12}$$

By a direct calculation we see that if $F \geq F_{\rm cr}$, then

$$\rho_{\rm as} = (1-p-q)\rho_\beta + p\,|a\rangle\langle a| + q\,|\psi\rangle\langle\psi|, \tag{III.13}$$

where

$$\begin{aligned}
p &= \left(1+\frac{c}{u}\right)F - \frac{c}{u},\\
q &= \frac{|z|\,(a+d)}{u\sqrt{ad}}\,(1-F).
\end{aligned} \tag{III.14}$$

The state ρ_β is a Gibbs state

$$\rho_\beta = \frac{e^{-\beta H_{\rm a}}}{\mathrm{tr}\, e^{-\beta H_{\rm a}}} \tag{III.15}$$

for the Hamiltonian $H_{\rm a} = H_0 + H_1$ with

$$H_0 = \frac{\omega_0}{2}\sum_{j=A,B}\sigma_3^j, \quad H_1 = \frac{\omega_1}{2}\left(\mathbb{1}\otimes\mathbb{1} + \sigma_3^A\otimes\sigma_3^B\right),$$

the inverse temperature

$$\beta = \frac{1}{2\omega_0}\ln\frac{d}{a}, \tag{III.16}$$

and the frequency

$$\omega_1 = \frac{2\omega_0}{\ln d/a}\ln\frac{c}{\sqrt{ad}-|z|}. \tag{III.17}$$

Moreover, the pure state $|\psi\rangle$ is given by

$$|\psi\rangle = \sqrt{\frac{a}{a+d}}\,|0\rangle_A\otimes|0\rangle_B + e^{i\phi}\sqrt{\frac{d}{a+d}}\,|1\rangle_A\otimes|1\rangle_B, \tag{III.18}$$

where $\phi = \arg z$.

The formula (III.13) is a generalization of the equation (III.9) as well as the corresponding representation of $\rho_{\rm as}$ by the thermal generalization of Werner states in the case of thermal reservoir [12]. Observe also that for $F < F_{\rm cr}$, the asymptotic state cannot be expressed as the mixture (III.13) but in contrast to the purely thermal case, the states $\rho_{\rm as}$ can be entangled even if $F < F_{\rm cr}$. We will study this problem in the next section.

IV. ASYMPTOTIC ENTANGLEMENT

For the characterization of entanglement of the asymptotic state $\rho_{\rm as}$ we use Wootters concurrence [20] defined for any two-qubit state ρ as

$$C(\rho) = \max\left(0, \sqrt{\lambda_1}-\sqrt{\lambda_2}-\sqrt{\lambda_3}-\sqrt{\lambda_4}\right), \tag{IV.1}$$

where $\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4$ are the eigenvalues of the matrix $\rho\widetilde{\rho}$ with $\widetilde{\rho}$ given by

$$\widetilde{\rho} = \sigma_2\otimes\sigma_2\,\overline{\rho}\,\sigma_2\otimes\sigma_2,$$

where $\overline{\rho}$ denotes complex conjugation of the matrix ρ. For the states in the "X" form, concurrence is given by the function

$$C(\rho) = \max\left(0, C_1, C_2\right), \tag{IV.2}$$

with

$$\begin{aligned} C_1 &= 2\left(|\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}\right),\\ C_2 &= 2\left(|\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}\right). \end{aligned} \tag{IV.3}$$

A. Entanglement of the asymptotic state $\rho_{\rm u}$

Let us start with spatially separated atoms which have the unique asymptotic state $\rho_{\rm u}$. Its concurrence is given by

$$C(\rho_{\rm u}) = 2\max\left(0, \frac{|z_0|-c_0}{u_0}, \frac{|b_0|-\sqrt{a_0 d_0}}{u_0}\right). \tag{IV.4}$$

Analysis of this function in the general case of broadband squeezed reservoir is involved, so we focus on the case of minimum - uncertainty squeezed states and consider (IV.4) as the function of squeezed field intensity N, for fixed values of parameters $\widehat{\gamma}$ and δ. We plot this function in FIG.1 for different values of detuning. It is evident that there is a range of values of mean photon number N for which the asymptotic concurrence is positive. Observe that the maximum of $C(\rho_{\rm u})$ appears for rather small values of N and the non - zero detuning diminishes the production of entanglement.

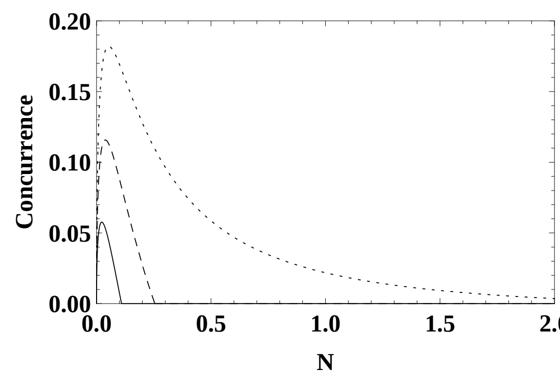

FIG. 1: Entanglement of the state $\rho_{\rm u}$ as a function of N for $\widehat{\gamma} = 0.85$ and different values of detuning: $\delta = 0$ (dotted curve); $\delta = 0.5$ (dashed curve) and $\delta = 1$ (solid curve).

B. Entanglement of the states $\rho_{\rm as}$

The properties of the concurrence of $\rho_{\rm as}$ as a function of initial fidelity can be studied in more details. Notice that for these states we have

$$C_1 = \left(\frac{c-2|z|}{u} - 1\right)F - \frac{c-2|z|}{u}. \tag{IV.5}$$

Define

$$F_1 = \max\left(0, \frac{c-2|z|}{c-2|z|-u}\right). \tag{IV.6}$$

If $F_1 > 0$, then

$$C_1 > 0 \quad \text{for} \quad 0 \le F < F_1.$$

On the other hand

$$C_2 = 2\left(\left|(1-F)\frac{c}{2u} - \frac{F}{2}\right| - (1-F)\frac{\sqrt{ad}}{u}\right). \tag{IV.7}$$

Notice that if $F < F_{\rm cr}$, then

$$(1-F)\,\frac{c}{2u} - \frac{F}{2} > 0,$$

and

$$C_2 = \left(\frac{2\sqrt{ad}-c}{u} - 1\right) F - \frac{2\sqrt{ad}-c}{u}. \tag{IV.8}$$

Since

$$\frac{2\sqrt{ad}-c}{u} - 1 < 0,$$

so

$$C_2 < 0 \quad \text{when} \quad F < F_{\rm cr}.$$

Let now $F \geq F_{\rm cr}$, then

$$C_2 = \left(1 + \frac{c+2\sqrt{ad}}{u}\right) F - \frac{c+2\sqrt{ad}}{u}. \tag{IV.9}$$

Define

$$F_2 = \frac{c+2\sqrt{ad}}{c+2\sqrt{ad}+u} \tag{IV.10}$$

Form the equation (IV.9) we see that

$$C_2 > 0 \quad \text{when} \quad F > F_2.$$

On the other hand, direct calculations show that

$$F_2 \geq F_{\rm cr} \quad \text{and} \quad F_1 \leq F_2.$$

Taking into account the above results we arrive at the conclusion that depending on the initial fidelity F, the asymptotic state $\rho_{\rm as}$ is entangled for all $F \in [0, F_1) \cup (F_2, 1]$ (provided $F_1 > 0$) and separable for $F \in [F_1, F_2]$ (see FIG.2). The asymptotic concurrence reads

$$C(\rho_{\rm as}) = \begin{cases} C_1, & 0 \leq F < F_1 \\ C_2, & F_2 < F \leq 1 \end{cases} \tag{IV.11}$$

with C_1 and C_2 given by equations (IV.5) and (IV.9), respectively. This general result covers also the special cases of vacuum reservoir where $F_2 = 0$, thermal reservoir with $F_1 = 0$ and $F_2 > 0$ and minimum-uncertainty squeezed reservoir, where $F_1 = F_2$. It is worth to stress that the creation of the asymptotic states with non-zero entanglement from the initial states with small or even zero fidelity is only possible when the reservoir is in a squeezed state. Let us discuss this point in more details in a special case of atoms which are in resonance with minimum - uncertainty radiation field. In this case

$$F_1 = F_2 = \frac{2\sqrt{N(N+1)}}{2\sqrt{N(N+1)} + (1+2N)}, \tag{IV.12}$$

and

$$C(\rho_{\rm as}) = \begin{cases} -(1+C_0)F + C_0, & F < F_1 \\ (1+C_0)F - C_0, & F > F_1 \end{cases} \tag{IV.13}$$

with

$$C_0 = 2\,\frac{\sqrt{N(N+1)}}{1+2N}. \tag{IV.14}$$

For all initial states with zero fidelity, we obtain pure entangled state (III.10) with concurrence equal to C_0. Notice that in the limit of maximal squeezing, this state becomes maximally entangled. For pure product states

$$|\Psi\rangle = |\varphi\rangle \otimes |\psi\rangle \tag{IV.15}$$

the fidelity is given by the formula

$$F = \frac{1}{2}\left(1 - |\langle\varphi|\psi\rangle|^2\right), \tag{IV.16}$$

so the zero fidelity corresponds, for example, to the case of two atoms prepared in the same initial states. This leads to the remarkable result: the interaction with squeezed reservoir will entangle two atoms which are initially in the ground state $|g\rangle = |0\rangle_A \otimes |0\rangle_B$. The analogous phenomenon cannot occur

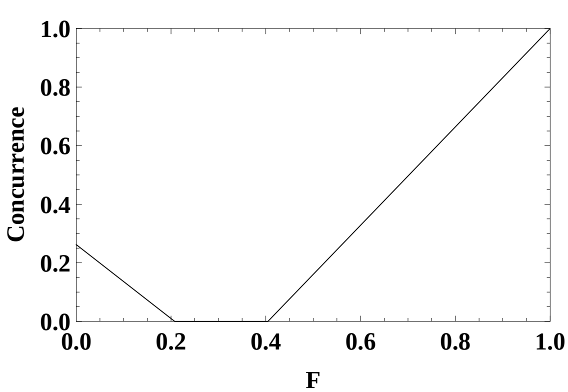

FIG. 2: Asymptotic entanglement versus fidelity for minimum - uncertainty squeezed reservoir with $N = 1$ and detuning $\delta = 0.8$

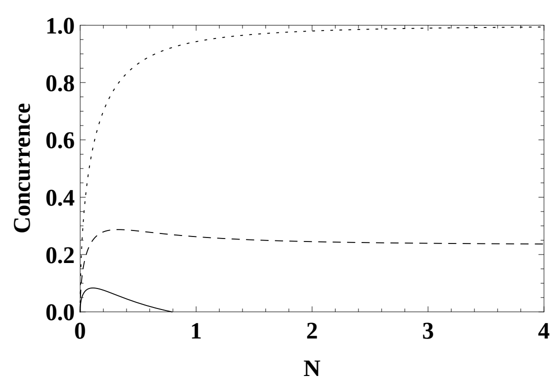

FIG. 3: Asymptotic entanglement of initial states with $F = 0$ as a function of N, for different values of detuning: $\delta = 0$ (dotted curve); $\delta = 0.8$ (dashed curve) and $\delta = 2$ (solid curve)

when the photon field is in the vacuum or thermal state. Notice also that for non - zero detuning, the asymptotic state is no longer pure and the production of stationary entanglement is less effective (FIG.3).

V. CONCLUSIONS

We have investigated the dynamics of two - level atoms interacting with the photon reservoir in a broadband squeezed vacuum state. Time evolution of the system crucially depend on the relative distance between the atoms. When the atoms are spatially separated, there is a unique asymptotic state, which can be entangled in contrast to the analogous asymptotic state for the thermal reservoir. In the case of small interatomic distance, there are nontrivial asymptotic states ρ_{as} which are parametrized by the fidelity F and the parameters N and M characterizing the squeezing. The states ρ_{as} depend also on the detuning between atomic transition frequency and carrier frequency of the photon field. For the values of F above the threshold fidelity F_2, the states ρ_{as} are entangled. Non - zero entanglement can also occur for small values of F or even if $F = 0$, and this possibility is a unique feature of the squeezed reservoir. When the atoms are in resonance with the photon field and $|M| = \sqrt{N(N+1)}$, the asymptotic state corresponding to $F = 0$ is the pure entangled state known as two - atom squeezed state.

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